Queen Mary University of London

Mean Curvature Flow and Related Topics

Workshop at the School of Mathematical Sciences
Queen Mary University of London
July 4–8, 2022

Timetable, Titles, and Abstracts

Monday

09:30 – 10:00	Welcome
10:00 – 11:00	Talk 1: Neshan Wickramasekera
11:00 – 12:00	Talk 2: Mario Schulz
	Lunch break
13:45 – 14:45	Talk 3: Stephen Lynch
14:45 – 15:45	Talk 4: Susanna Risa
	Coffee break in the *Social Hub*
16:30 – 17:30	Talk 5: Ben Lambert
17:30	Wine reception in the *Social Hub*

Tuesday

09:00 – 10:00	Talk 6: Tobias Lamm
10:00 – 11:00	Talk 7: Shengwen Wang
	Coffee break in the *Common Room*
11:30 – 12:30	Talk 8: Or Hershkovits
	Lunch break
14:15 – 15:15	Talk 9: Gianmichele Di Matteo
	Coffee break in the *Common Room*
16:00 – 17:00	Talk 10: Niels Møller

Wednesday

09:00 – 10:00	Talk 11: Jacob Bernstein
10:00 – 11:00	Talk 12: Elena Mader-Baumdicker
	Coffee break in the *Social Hub*
11:30 – 12:30	Talk 13: Keaton Naff

Thursday

09:00 – 10:00	Talk 14: Peter Topping
10:00 – 11:00	Talk 15: Felix Schulze
	Coffee break in the *Social Hub*
11:30 – 12:30	Talk 16: Panagiotis Gianniotis
	Lunch break
14:15 – 15:15	Talk 17: Sylvain Maillot
	Coffee break in the *Social Hub*
16:00 – 17:00	Talk 18: Mariel Saez
18:30	Social dinner at *The Coborn*

Friday

09:00 – 10:00	Talk 19: Jonathan Zhu
10:00 – 11:00	Talk 20: Alec Payne
11:00	Coffee in the *Social Hub* and good-bye

All talks are in the Maths Lecture Theatre. Coffee breaks and wine reception are either in the Social Hub (floor -1) or the Common Room (floor 5) as indicated on the time table.

Talk 1: **Neshan Wickramasekera** (Cambridge University)

Higher multiplicity in stable codimension 1 stationary varifolds

Abstract: The talk will be based on recent joint work with Paul Minter (Cambridge). The work develops a method to analyse the behaviour of a large class of stable, stationary codimension 1 varifolds near their branch points, i.e. singular points where one tangent cone is a hyperplane of some integer multiplicity ≥ 2. A main outcome is a local description of the varifold near a branch point, including uniqueness of tangent cones. Understanding branch points, or ruling them out, is a well-known difficulty that one faces, for instance, when proving existence theorems for minimal hypersurfaces or when studying weak limits of regular minimal hypersurfaces under area bounds. For the problem of codimension 1 area minimisation, we have known since the 1970's, essentially by De Giorgi's work, that branch points do not exist if we minimise in the class of integral currents; for currents that are mod p minimisers however, it has remained an open problem as to what one can say about the current near such points (for even p, which is the only relevant case) except in the cases $p = 2$ or $p = 4$ which were settled in the 1970's by the Allard regularity theory and a theorem of White respectively. A direct consequence of our theory is to settle this question for arbitrary (even) p.

Talk 2: **Mario Schulz** (Universität Münster)

Noncompact self-shrinkers for mean curvature flow

Abstract: In his lecture notes on mean curvature flow, Ilmanen conjectured the existence of noncompact self-shrinkers with arbitrary genus. We employ min-max techniques to give a rigorous existence proof for these surfaces. Conjecturally, the self-shrinkers that we obtain have precisely one (asymptotically conical) end. We confirm this for large genus via a precise analysis of the limiting object of sequences of such self-shrinkers for which the genus tends to infinity. This is joint work with Reto Buzano and Huy The Nguyen.

Talk 3: **Stephen Lynch** (Universität Tübingen)

A gradient estimate for two-convex immersions

Abstract: Gradient estimates for the second fundamental form have proven to be a powerful tool for controlling the geometry of singularities and performing surgery on geometric flows. Brendle and Huisken introduced a fully nonlinear flow for two-convex hypersurfaces in Riemannian manifolds, and proved such a gradient estimate for solutions which start from the boundary of a region. We will describe a completely different approach to proving this estimate which applies even if the initial hypersurface is only immersed. It follows that one can perform surgery in this setting, and so classify two-convex immersions in a natural class of ambient spaces.

Talk 4: **Susanna Risa** (Università degli Studi di Napoli)

Round point convergence and convex ancient solution for flows with high degree of homogeneity

Abstract: Under Mean Curvature Flow, a compact convex hypersurface converges to a point in finite time, while its shape becomes more and more spherical. This fundamental result of Huisken has been subsequently generalised to fully nonlinear flows and it is fairly well established, under broad assumptions, for speeds which are homogeneous in the principal curvatures and with degree of homogeneity equal to one. In contrast, what happens for flows having degree of homogeneity greater than one is far less understood, unless the starting hypersurface is strictly pinched or for very specific speeds. In this talk, we will discuss convex hypersurfaces which are axially stretched - a particular case of rotational symmetry - and show how they converge to round points in finite time under a large class of speeds with high degree of homogeneity. Afterwards, we will show how a nonspherical ancient solution, an ancient Ovaloid, can be constructed in this setting. Joint work with Carlo Sinestrari.

Talk 5: **Ben Lambert** (University of Leeds)

Alexandrov immersed mean curvature flow

Talk 6: **Tobias Lamm** (Karlsruher Institut für Technologie)
Ricci flow of $W^{2,2}$-metrics in four dimensions
Abstract: In this talk we construct solutions to the Ricci-DeTurck flow in four dimensions for $W^{2,2}$-initial metrics. A Ricci-flow related solution is also constructed whose initial value is isometric in a weak sense to the initial value of the Ricci-DeTurck flow. This is a joint work with Miles Simon.

Talk 7: **Shengwen Wang** (Warwick University)
A Brakke type regularity theorem for the Allen-Cahn flow
Abstract: I will present an analogue of the Brakke's local regularity theorem for the ϵ parabolic Allen-Cahn equation. In particular, we show uniform $C_{2,\alpha}$ regularity for the transition layers converging to smooth mean curvature flows as ϵ tend to 0 under the almost unit-density assumption. This can be viewed as a diffused version of the Brakke regularity for the limit mean curvature flow. This talk is based on joint work with Huy Nguyen.

Talk 8: **Or Hershkovits** (Hebrew University of Jerusalem)
The Hopf Lemma for Brakke flows
Abstract: In this talk, I will describe a variant of the classical Hopf lemma, that allows to show regularity (and non-vanishing angle) at (boundary) intersection points of two Brakke flows which are disjoint in a half of a parabolic ball. This Hopf Lemma can be used in the moving plane method, allowing to prove symmetry and regularity in tandem. This is based on a joint work with Kyeongsu Choi, Robert Haslhofer and Brian White.

Talk 9: **Gianmichele Di Matteo** (Karlsruher Institut für Technologie)
A local singularity analysis for the Ricci flow
Abstract: In this talk, we will describe a refined local singularity analysis for the Ricci flow developed jointly with R. Buzano. The key idea is to investigate blow-up rates of the curvature tensor locally, near a singular point. Then we will show applications of this theory to Ricci flows with scalar curvature bounded up to the (possibly) singular time.

Talk 10: **Niels Martin Møller** (GeoTop, U Copenhagen)
On the entropy spectrum of embedded self-shrinkers with symmetries
Abstract: We consider complete shrinking solitons for the mean curvature flow of hypersurfaces in $\mathbb{R}^{n+1}$. Using comparison geometry, we find an explicit universal constant bounding the entropies of all such embedded self-shrinkers with rotational symmetry. As applications, we prove within this class smooth compactness and, via the Lojasiewicz-Simon inequality, finiteness of the entropy spectrum. This is joint work with Ali Muhammad and John Ma.

Talk 11: **Jacob Bernstein** (Johns Hopkins University)
Density lower bounds for topologically non-trivial regular minimal cones in low dimensions
Abstract: Using the Hardt-Simon foliation and mean curvature flow, Ilmanen-White were able to establish that any area-minimizing regular cone that was topologically non-trivial has density at least $\sqrt{2}$, partially verifying a conjecture of Solomon. In low dimensions there are no non-trivial area-minimizing cones. Using some recently established properties of low-entropy self-expanders of mean curvature flow, we establish the same bound for any topologically non-trivial regular minimal cones in these dimensions. This is joint work with L. Wang.

Talk 12: **Elena Mäder-Baumdicker** (Technische Universität Darmstadt)

Non-local estimates for the volume preserving mean curvature flow and applications

Abstract: The volume preserving mean curvature flow (VPMCF) deforms a hypersurface along the (negative) gradient of the area functional while keeping the enclosed volume fixed. Due to the global constraint the VPMCF behaves quite differently compared to the mean curvature flow. I will point our differences and report on recent work with Ben Lambert about the VPMCF in the closed, Euclidean setting. We were able to obtain estimates of non-local quantities that allow us to apply classical parabolic blowup procedures. Furthermore, we found a property that is preserved along the VPMCF.

Talk 13: **Keaton Naff** (Columbia University)

Immersed mean curvature flows with noncollapsed singularities

Abstract: In the mean curvature flow of hypersurfaces, noncollapsing has proven to be a powerful and useful assumption when studying singularities and high curvature regions. In particular, the assumption of noncollapsing has been used to prove a wide range of local a priori estimates, and has led to classification results for certain classes of singularity models. Less is known for immersed mean-convex flows. In this talk, I would like to survey recent results and discuss outstanding conjectures for immersed mean-convex flows that begin to bridge the gap between the embedded and immersed mean-convex settings. The talk is based on joint work with S. Brendle and ongoing work with S. Lynch.

Talk 14: **Peter Topping** (Warwick University)

Hamilton's pinching conjecture

Abstract: The Bonnet-Myers theorem tells us that a uniform positive lower bound on the Ricci curvature of a manifold has topological implications. Richard Hamilton proposed a scale invariant version of this theorem. I will give an introduction to the problem and describe some of the interesting work that has been developed over the years in an attempt to solve it. This year Hamilton's conjecture has been solved as a result of new work of Deruelle-Schulze-Simon and of M.C.Lee and myself. I will give a simplified overview of a rough strategy and describe our contribution. In a subsequent talk Felix Schulze will describe their breakthrough on a different part of the problem.

Talk 15: **Felix Schulze** (Warwick University)

Initial stability estimates for Ricci flow and three dimensional Ricci-pinched manifolds

Abstract: Following Peter Topping's introduction to Hamilton's pinching conjecture and the fundamental contribution of Lee-Topping to its recent full resolution we will explain how an initial stability estimate for Ricci flows which start at a possibly non-smooth metric space enters as a central new tool in our contribution. More precisely we will discuss that if the initial metric space is Reifenberg and locally bi-Lipschitz to Euclidean space, then two solutions to the Ricci flow whose Ricci curvature is uniformly bounded from below and whose curvature is bounded by ct^{-1} converge to one another at an exponential rate once they have been appropriately gauged. Our work is joint with A. Deruelle and M. Simon.

Talk 16: **Panagiotis Gianniotis** (University of Athens)

An isometric flow of G_2 structures

Abstract: A G_2 structure on a 7-manifold is a three form that determines, in a non-linear way, a Riemannian metric. G_2 structures which are parallel with respect to the associated Levi-Civita connection induce metrics which are automatically Ricci flat with holonomy contained in the Lie group G_2. Parallel G_2 structures can be considered as the optimal such structures on a given smooth 7-manifold. There are, however, obstructions to their existence, and despite the construction of many examples, there is at the moment no conjecture regarding which smooth 7-manifolds admit holonomy G_2 metrics. On the other hand, any Riemannian metric on a manifold admitting G_2 structures is induced by many *isometric* G_2 structures, and a natural question is to find whether there exists an optimal representative in a given isometric class. In this talk I will discuss a geometric flow approach to this problem, initially proposed by Grigorian, and present joint work with Dwivedi and Karigiannis in which we develop the foundational theory for this flow.

Talk 17: **Sylvain Maillot** (Université de Montpellier)

Mean curvature flow and Heegaard surfaces in lens spaces

Abstract: Lens spaces $L(p, q)$ are a family of closed 3-manifolds indexed by two coprime integers. They can be described as quotients of the 3-sphere by free isometric actions of cyclic groups; hence they carry riemannian metrics of constant positive sectional curvature. Alternatively, they can be obtained by gluing together two solid tori along their common boundary, which is called a Heegaard torus. Our main theorem is as follows: fix a metric of constant sectional curvature 1 on $L(p, q)$, and denote by $\mathcal{M}_{H>0}(p, q)$ the moduli space of Heegaard tori in $L(p, q)$ that have positive mean curvature. If $q \cong \pm 1 \mod p$, then $\mathcal{M}_{H>0}(p, q)$ is path-connected. Otherwise it has exactly two path-components. This is work in progress, joint with Reto Buzano.

Talk 18: **Mariel Saez** (Pontificia Universidad Católica de Chile)

Uniqueness of entire graphs evolving by mean curvature flow

Abstract: In this work we study the uniqueness of graphical mean curvature flow with locally Lipschitz initial data. We first prove that rotationally symmetric entire graphs are unique, without any further assumptions. Our methods also give an alternative simple proof of uniqueness in the one dimensional case. In the general case, we establish the uniqueness of entire proper graphs that satisfy a uniform lower bound on the second fundamental form. The latter result extends to initial conditions that are proper graphs over subdomains of Rn. A consequence of our result is the uniqueness of convex entire graphs, which allows us to prove that Hamilton's Harnack estimate holds for mean curvature flow solutions that are convex entire graphs. This is joint work with P. Daskalopoulos.

Talk 19: **Jonathan Zhu** (Princeton University)

Prescribed-point area estimates in space forms

Abstract: We discuss sharp area estimates for minimal submanifolds that pass through a prescribed point in a geodesic ball in a space form. The corresponding estimate in Euclidean space was first conjectured by Alexander, Hoffman and Osserman in 1974, and was previously proven in full generality by Brendle and Hung.

Talk 20: **Alec Payne** (Duke University)

Mass drop and multiplicity in mean curvature flow

Abstract: Mean curvature flow can be continued through singularities using Brakke flow, a weak solution to the flow. Brakke flow is defined with an inequality which makes it tantamount to a subsolution to mean curvature flow. In this talk, we will present results relating the multiplicity one conjecture to uniqueness problems for Brakke flow. First, we will discuss how continuity of mass along the flow is equivalent to a weak version of the multiplicity one conjecture. Then, we will discuss how Brakke flows achieve equality in their defining inequality when they have generic singularities, i.e. singularities of mean convex type. This uses an analysis of worldlines in Brakke flows, analogous to the theory of singular Ricci flows, and answers a conjecture of Ilmanen in an important case.